**MATH 2050B Mathematical Analysis I**
**2023-24 Term 1**
**Problem Set 10**

*due on Dec 1, 2023 (Friday) at 11:59PM*

**Instructions**: You are allowed to discuss with your classmates or seek help from the TAs but you are required to write/type up your own solutions. You can either type up your assignment or scan a copy of your written assignment into ONE PDF file and submit through Gradescope on/before the due date. Please remember to write down your name and student ID. **No late homework will be accepted.** All the exercises below are taken from the textbook.

**Required Readings**: Chapter 5.3, 5.4

**Optional Readings**: Chapter 5.6

## Problems to hand in

Section 5.3: Exercise # 2, 3, 6, 17

Section 5.4: Exercise # 2, 6, 8, 12

## Suggested Exercises

Section 5.3: Exercise # 1, 4, 11, 12, 15, 16

Section 5.4: Exercise # 1, 3, 4, 5, 7, 9, 10, 11, 14, 15

## Challenging Exercises (optional)

1. Section 5.3: Exercise # 13
2. Suppose $f : \mathbb{R} \to \mathbb{R}$ is a function such that
$$\lim_{h \to 0} \big( f(x+h) - f(x-h) \big) = 0$$
for every $x \in \mathbb{R}$. Is $f$ necessarily a continuous function on $\mathbb{R}$?
3. Let $f : [0,1] \to \mathbb{R}$ be a continuous function such that $f([0,1]) = [0,1]$. Prove that there exists some $x_0 \in [0,1]$ such that $f(x_0) = x_0$.

4. Section 5.4: Exercise # 13, 16
5. Let $f : \mathbb{Q} \to \mathbb{R}$ be a uniformly continuous function. Prove that there exists a continuous function $g : \mathbb{R} \to \mathbb{R}$ such that $g(x) = f(x)$ for all $x \in \mathbb{Q}$.
6. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $|f(x)| \leq 1$ for all $x \in \mathbb{R}$. Suppose $g : [-1, 1] \to \mathbb{R}$ is a continuous injective function. Prove that $f$ is (uniformly) continuous if $g \circ f$ is (uniformly) continuous.